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STABILIZATION OF THE MOTION OF A DYNAMIC SYSTEM UNDER CONDITIONS OF UNCERTAINTY*

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The problem of the stabilization of the unperturbed motion of a dynamic system when there is incomplete information about the system parameters is considered. The solution is sought by Lyapunov's second method in the class of dynamic controllers and generalizes the result obtained in /1/ to controlled dynamic systems. Similar control problems were considered, in particular, in /2/.

The solution is used to stabilize the permanent rotation of a rigid body by a controlling moment with zero x-component /3/.

1. *Statement of the problem.* Consider a controlled dynamic system (a controlled plant)

$$\mathbf{x}^{\cdot} = \mathbf{f}(\mathbf{x}, \mathbf{u}, \xi), \ \mathbf{f}(0, 0, \xi) = 0 \qquad (1.1)$$
$$\mathbf{x} \in R_n, \ \mathbf{u} \in U \subset R_m, \ \xi \in R_s$$

where ξ is the s-dimensional vector of unknown parameters.

In the domain

$$P = \{\mathbf{x}, \xi: \|\mathbf{x}\| < \nu_1, \|\xi\| < \nu_2\} \qquad (1.2)$$

(ν_1 and ν_2 are positive constants), the functions $f_i(\mathbf{x}, \mathbf{u}, \xi)$ $(i = 1, \ldots, n)$ are continuous together with their partial derivatives with respect to $x_1, \ldots, x_n, \xi_1, \ldots, \xi_s$ and there exists a constant $\nu_3 > 0$ such that

$$|\partial^2 f_i/\partial\xi_j\partial\xi_k| < \nu_3 \ (i = 1, \ldots, n; \ j, k = 1, \ldots, s)$$

It is required to a find a control u which ensures asymptotic stability in the domain (1.2) of the equilibrium

$$\mathbf{x} = 0 \qquad (1.3)$$

of system (1.1).

2. *Stabilization of a controlled system with unknown parameters.* Assume that the stabilization problem with unknown vector ξ is solved by the control

$$\mathbf{u} = \mathbf{u}_*(\mathbf{x}, \xi), \ \mathbf{u}_*(0, \xi) = 0 \qquad (2.1)$$

which corresponds in the domain (1.2) to a positive-definite function W satisfying the equation

$$(\partial V_0/\partial\mathbf{x})\cdot\mathbf{f}(\mathbf{x}, \mathbf{u}_*, \xi) = -W(\mathbf{x}, \xi);$$

the function $V_0(\mathbf{x})$ is positive-definite in this domain.

We will also assume that system (1.1) is identifiable on the unperturbed trajectory (1.3) /4/.

The solution of this problem with unkown vector ξ is sought in the class of dynamic controllers /2/:

$$\mathbf{y}^{\cdot} = \mathbf{\Phi}(\mathbf{x}, \mathbf{y}, \mathbf{u}, \eta), \quad \eta^{\cdot} = \mathbf{g}(\mathbf{x}, \mathbf{y}, \eta), \quad \mathbf{u} = \mathbf{u}(\mathbf{x}, \eta) \qquad (2.2)$$

where η is the s-dimensional estimate vector of the plant parameters ξ, and $\mathbf{g}(\mathbf{x}, \mathbf{y}, \eta)$ is the required vector function.

Specifying the right-hand side of the first equation in (2.2) in the form /1/

$$\mathbf{\Phi}(\mathbf{x}, \mathbf{y}, \mathbf{u}, \eta) = A(\mathbf{y} - \mathbf{x}) + \mathbf{f}(\mathbf{x}, \mathbf{u}, \eta)$$

(A is a stable $n \times n$ matrix), we note that the difference $\mathbf{e} = \mathbf{x} - \mathbf{y}$ between the plant state vector $\mathbf{x}$ and the state vector of the parameter tracking system $\mathbf{y}$ satisfies the equation

$$\mathbf{e}^{\cdot} = A\mathbf{e} + \mathbf{f}(\mathbf{x}, \mathbf{u}, \xi) - \mathbf{f}(\mathbf{x}, \mathbf{u}, \eta) = A\mathbf{e} + \frac{\partial\mathbf{f}}{\partial\xi}(\mathbf{x}, \mathbf{u}, \eta)\alpha + \mathbf{h}(\mathbf{x}, \mathbf{u}, \eta, \alpha), \quad \alpha = \xi - \eta \qquad (2.3)$$
$$\|\mathbf{h}(\mathbf{x}, \mathbf{u}, \eta, \alpha)\|/\|\alpha\| \to 0 \quad \text{for } \|\alpha\| \to 0$$

Here we have used the previously listed properties of the vector function $\mathbf{f}(\mathbf{x}, \mathbf{u}, \xi)$, which make it possible to write the difference of the last two terms on the right-hand side of Eq.(2.3) in the above form /1/.

Forming the control (2.1) from the estimate η of the vector ξ, we assume that the control

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can be represented in the form

$$\mathbf{u}(\mathbf{x}, \boldsymbol{\eta}) = \mathbf{u}_*(\mathbf{x}, \boldsymbol{\xi}) + Q(\boldsymbol{\xi})\boldsymbol{\alpha} + \mathbf{q}(\mathbf{x}, \boldsymbol{\alpha}, \boldsymbol{\xi}) \tag{2.4}$$

where $Q(\boldsymbol{\xi})$ is an $m \times s$ matrix and $\mathbf{q}(\mathbf{x}, \boldsymbol{\alpha}, \boldsymbol{\xi})$ is a vector function whose expansion in the variables $\mathbf{x}$ and $\boldsymbol{\alpha}$ starts with terms not lower than the second order.

Now consider the plant Eq.(1.1), system (2.3), the equation for the error $\boldsymbol{\alpha}$ of the estimated parameters

$$\boldsymbol{\alpha}^{\cdot} = -\mathbf{g}(\mathbf{x}, \mathbf{e}, \boldsymbol{\eta}) \tag{2.5}$$

and the control algorithm (2.4). The problem reduces to finding a vector function $\mathbf{g}(\mathbf{x}, \mathbf{e}, \boldsymbol{\eta})$ which ensures asymptotic stability of the equlibrium

$$\mathbf{x} = 0, \ \mathbf{e} = 0, \ \boldsymbol{\alpha} = 0 \tag{2.6}$$

of system (1.1), (2.3)-(2.5).

We will introduce the function

$$V(\mathbf{x}, \mathbf{e}, \boldsymbol{\alpha}) = V_0(\mathbf{x}) + \mathbf{e}'R\mathbf{e} + \boldsymbol{\alpha}'\Gamma^{-1}\boldsymbol{\alpha} \tag{2.7}$$

(R and Γ are symmetric $n \times n$ and $s \times s$ matrices, respectively) which is positive everywhere except at the point (2.6), where it vanishes. The derivative of V with respect to time, by virtue of the unperturbed system (1.1), (2.3)-(2.5) can be represented in the form

$$\begin{gathered} V^{\cdot} = -W(\mathbf{x}, \boldsymbol{\xi}) + 2\mathbf{x}'D\boldsymbol{\alpha} + \mathbf{e}'N\mathbf{e} + \\ 2\mathbf{e}'R(\partial\mathbf{f}/\partial\boldsymbol{\xi})\boldsymbol{\alpha} - 2\boldsymbol{\alpha}'\Gamma^{-1}\mathbf{g} + v(\mathbf{x}, \mathbf{e}, \boldsymbol{\alpha}) \\ N = A'R + RA \end{gathered} \tag{2.8}$$

where, as a result of the stability of the matrix A the symmetric matrix $N < 0$; the expansion of the function $v(\mathbf{x}, \mathbf{e}, \boldsymbol{\alpha})$ in the variables $\mathbf{x}, \mathbf{e}, \boldsymbol{\alpha}$ starts with terms of not lower than the third degree and $v(0, 0, \boldsymbol{\alpha}) = 0$. Note that the term $2\mathbf{x}'D\boldsymbol{\alpha}$ in (2.8) is attributable to the term $Q(\boldsymbol{\xi})\boldsymbol{\alpha}$ in the structure of the control $\mathbf{u}$.

Sign-definiteness of analytical functions is determined by the terms of lowest order in their expansion /5/. We can therefore judge the properties of the function (2.8) from the properties of the functions

$$\begin{gathered} V^{\circ}(\mathbf{x}, \mathbf{e}, \boldsymbol{\alpha}) = -W(\mathbf{x}, \boldsymbol{\xi}) + \mathbf{e}'N\mathbf{e} + \\ 2\mathbf{e}'[D\mathbf{x} + (\partial\mathbf{f}/\partial\boldsymbol{\xi})'R\mathbf{e} - \Gamma^{-1}\mathbf{g}] \end{gathered} \tag{2.9}$$

If we put

$$\mathbf{g}(\mathbf{x}, \mathbf{e}, \boldsymbol{\eta}) = \Gamma(\partial\mathbf{f}/\partial\boldsymbol{\xi})'R\mathbf{e} + \Gamma D\mathbf{x} \tag{2.10}$$

then the right-hand side of (2.9) as a function of the vector $\mathbf{z} = \{\mathbf{x}', \mathbf{e}', \boldsymbol{\alpha}'\}$ is negative definite, because it is zero not only at the point (2.6) but also on the set

$$Z = \{\mathbf{z}: \mathbf{x} = 0, \ \mathbf{e} = 0, \ \boldsymbol{\alpha} \neq 0\}$$

The set Z does not contain entire trajectories of the system (1.1), (2.3)-(2.5), and therefore by Krasovskii's theorem /6/ the control $\mathbf{u} = \mathbf{u}(\mathbf{x}, \boldsymbol{\eta})$ and the identification law (2.10) ensure asymptotic stability of the unperturbed motion (2.6) of system (1.1), (2.3)-(2.5).

Note in conclusion that the system of equations (the dynamic controller)

$$\begin{gathered} \mathbf{y}^{\cdot} = A(\mathbf{y} - \mathbf{x}) + \mathbf{f}(\mathbf{x}, \mathbf{u}(\mathbf{x}, \boldsymbol{\eta}), \boldsymbol{\eta}), \ \boldsymbol{\eta}^{\cdot} = \Gamma(\partial\mathbf{f}/\partial\boldsymbol{\xi})'R(\mathbf{x} - \mathbf{y}) + \\ \Gamma D\mathbf{x}, \ \mathbf{u} = \mathbf{u}(\mathbf{x}, \boldsymbol{\eta}) \end{gathered}$$

not only solves the stabilization problem but also determines the vector $\boldsymbol{\xi}$ of unknown plant parameters.

3. *Stabilization of the permanent rotation of a rigid body.* We will introduce two right-handed orthogonal systems of coordinates: the coordinate system xyz rigidly attached to the body and the system $x_*y_*z_*$ of principal central axes of inertia of the body.

We will assume that the motion of the rigid body is observed in the basis xyz and is described by Euler's dynamic equations

$$J\boldsymbol{\omega}^{\cdot} + \boldsymbol{\omega} \times J\boldsymbol{\omega} = \mathbf{M}, \ \boldsymbol{\omega} = \{\omega_x, \omega_y, \omega_z\} \tag{3.1}$$

The controlling moment $\mathbf{M}$ has the structure $\mathbf{M} = \{0, M_y, M_z\}$.

The matrix of inertia J is related to the matrix of inertia $J_* = \operatorname{diag}\{J_1, J_2, J_3\}$ (to fix our ideas, let $J_1 < J_2 \leqslant J_3$) in the basis $x_*y_*z_*$ by the equality

$$J = BJ_*B', \ B = \{\beta_{ij}\} \ (i, j = 1, 2, 3) \tag{3.2}$$

(B is the matrix of direction cosines).

Assume that the angular velocity $\omega_x = \Omega\ (|\omega_y|, |\omega_z| \ll \Omega)$ is imparted to the rigid body with known moments of inertia in the system $x_* y_* z_*$. It is required to stabilize the permanent rotation of the body about the x_* axis with the angular velocity Ω_* for an unkown attitude of the principal central axis of inertia x_* of the rigid body in the xyz system. In the xyz basis, this corresponds to the motion

$$\boldsymbol{\omega} = \boldsymbol{\omega}_* = \Omega_* \boldsymbol{\xi}, \quad \boldsymbol{\xi} = \{\beta_{11}, \beta_{21}, \beta_{31}\} \tag{3.3}$$

We will now introduce new variables X_i by the formula

$$\boldsymbol{\omega} = \mathbf{X} + \Omega_* \boldsymbol{\xi}, \quad \mathbf{X} = \{X_i\} \quad (i = 1, 2, 3) \tag{3.4}$$

and write the equation of perturbed motion of the rigid body in the form

$$J\mathbf{X}^{\cdot} + \Omega_* \boldsymbol{\xi} \times J\mathbf{X} + \mathbf{X} \times J(\mathbf{X} + \Omega_* \boldsymbol{\xi}) = \mathbf{M} \tag{3.5}$$

Parametrize the matrix B of direction cosines using the variables

$$\tau_1 = \beta_{21}, \quad \tau_2 = \beta_{31}, \quad \tau_3 = \beta_{23}$$

and represent it in the form

$$B = \left\| \begin{matrix} \gamma_1 & \tau_2\tau_3 - \tau_1\gamma_3 & T \\ \tau_1 & \gamma_2 & \tau_3 \\ \tau_2 & \tau_1 T - \tau_3\gamma_1 & \gamma_3 \end{matrix} \right\|, \quad \begin{matrix} \gamma_1 = (1 - \tau_1^2 - \tau_2^2)^{1/2} \\ \gamma_2 = (1 - \tau_1^2 - \tau_3^2)^{1/2} \\ \gamma_3 = (1 - \tau_3^2 - T^2)^{1/2} \end{matrix}$$

$$T = -(\tau_1\tau_3\gamma_1 + \tau_2\gamma_2)/(1 - \tau_1^2)$$

Note that along the unperturbed trajectory (3.3) the parameters τ_1 and τ_2 are identifiable, while the parameter τ_3 is unidentifiable. Because of the unidentifiability of τ_3, we may take an arbitrary value (e.g., zero) for this parameter.

Since the direction of the x_* axis in the xyz basis is determined by the parameters τ_1 and τ_2, we need to identify the vector ξ (or more precisely two components β_{21} and β_{31} of this vector).

As the tracking model for the parameter vector ξ of the rigid body, we use the system of equations

$$\mathbf{y}^{\cdot} = A(\mathbf{y} - \boldsymbol{\omega}) + J_+^{-1}(\mathbf{M} - \boldsymbol{\omega} \times J_+ \boldsymbol{\omega}) \tag{3.6}$$

where J_+ is the matrix of inertia of the body, calculated using the estimate η of the vector ξ.

To construct the identification algorithm (2.10), we calculate the matrix $\partial\mathbf{f}/\partial\boldsymbol{\xi} \equiv \| 0\, \partial\mathbf{f}/\partial\boldsymbol{\tau} \|$, $\boldsymbol{\tau} = \{\tau_1, \tau_2\}$ (0 is the 3×1 matrix of zeros) from the formula

$$\frac{\partial\mathbf{f}}{\partial\tau_i} = \frac{\partial J_+^{-1}}{\partial\tau_i}(\mathbf{M} - \boldsymbol{\omega} \times J_+\boldsymbol{\omega}) - J_+^{-1}\left(\boldsymbol{\omega} \times \frac{\partial J_+}{\partial\tau_i}\boldsymbol{\omega}\right) \quad (i = 1, 2)$$

where using (3.2)

$$\frac{\partial C_+}{\partial\tau_i} = \frac{\partial B}{\partial\tau_i} C_* B' + B C_* \frac{\partial B'}{\partial\tau_i}$$

$$(C_+ = J_+, J_+^{-1};\ C_* = J_*, J_*^{-1};\ i = 1, 2)$$

In /3/ the permanent rotation of a rigid body (3.3) with known vector ξ is stabilized by the control

$$\mathbf{M} = kQ(\boldsymbol{\xi})\boldsymbol{\omega}, \quad k < 0 \tag{3.7}$$

$$Q(\boldsymbol{\xi}) = \left\| \begin{matrix} 0 & 0 & 0 \\ -\beta_{11}\beta_{21} & 1 - \beta_{21}^2 & -\beta_{21}\beta_{31} \\ -\beta_{11}\beta_{31} & -\beta_{21}\beta_{31} & 1 - \beta_{31}^2 \end{matrix} \right\|$$

or in coordinate form

$$M_y = k(\omega_y - \beta_{21}(\boldsymbol{\xi}'\boldsymbol{\omega})), \quad M_z = k(\omega_z - \beta_{31}(\boldsymbol{\xi}'\boldsymbol{\omega}))$$

To solve the problem with unkown vector ξ, we construct the controlling moment $\mathbf{M}$ in the form (3.7), replacing the vector ξ in the matrix Q with its estimate $\eta = \xi - \alpha$. The control obtained in this way is written in the form

$$\mathbf{M} = kQ(\boldsymbol{\eta})\boldsymbol{\omega} = kQ(\boldsymbol{\xi})(\mathbf{X} + \Omega_*\boldsymbol{\alpha}) + \mathbf{m}(\mathbf{X}, \boldsymbol{\alpha}) \tag{3.8}$$

where $\mathbf{m}(\mathbf{X}, \boldsymbol{\alpha})$ is a vector function whose expansion in powers of the variables $\mathbf{X}$ and $\boldsymbol{\alpha}$ starts with terms of not less than second degree.

Let us now determine the structure of the identification law. Let

$$\mathbf{X} = B\mathbf{x},\ \mathbf{e} = B\mathbf{e}_*,\ \boldsymbol{\alpha} = B\boldsymbol{\alpha}_*,\ \mathbf{m} = B\mathbf{m}_*$$
$$\boldsymbol{\alpha}_* = \{\alpha_i^*\},\ \mathbf{m}_* = \{m_i^*\}\ (i = 1, 2, 3)$$

The system (1.1), (2.3)-(2.5), with the first equation in the form (3.5), is written in the $x_*y_*z_*$ basis, taking into account the control (3.8):

$$\begin{gathered} J_1x_1^{\cdot} = (J_2 - J_3)\, x_2x_3 - k\,[\beta_{11}\beta_{12}\,(x_2 + \Omega_*\alpha_2^*) + \\ \beta_{11}\beta_{13}\,(x_3 + \Omega_*\alpha_3^*)] + m_1^*\,(\mathbf{x},\ \boldsymbol{\alpha}_*) \\ J_2x_2^{\cdot} = (J_3 - J_1)\,(x_1 + \Omega_*)\, x_3 + k\,[(1 - \beta_{12}^2)\,(x_2 + \\ \Omega_*\alpha_2^*) - \beta_{12}\beta_{13}\,(x_3 + \Omega_*\alpha_3^*)] + m_2^*\,(\mathbf{x},\ \boldsymbol{\alpha}_*) \end{gathered} \tag{3.9}$$

$$\begin{gathered} J_3x_3^{\cdot} = (J_1 - J_2)\,(x_1 + \Omega_*)\, x_2 + k\,[-\beta_{12}\beta_{13}\,(x_2 + \\ \Omega_*\alpha_2^*) + (1 - \beta_{13}^2)\,(x_3 + \Omega_*\alpha_3^*)] + m_3^*\,(\mathbf{x},\ \boldsymbol{\alpha}_*) \end{gathered}$$

$$\begin{gathered} \mathbf{e}_*^{\cdot} = A_*\mathbf{e}_* + B'\,(\partial\mathbf{f}/\partial\xi)\, B\boldsymbol{\alpha}_* + \mathbf{h}_*,\ \boldsymbol{\alpha}_*^{\cdot} = \mathbf{g}_*\,(\mathbf{x},\ \mathbf{e}_*,\ \boldsymbol{\alpha}_*) \\ A_* = B'AB,\ \mathbf{g}_* = \{g_i^*\} = -B'\mathbf{g}\ (i = 1, 2, 3) \end{gathered} \tag{3.10}$$

The new variables Y_1 and Y_2, related to the variables $x_2, x_3, \alpha_2^*, \alpha_3^*$ by the equalities

$$Y_1 = x_2 + \Omega_*\alpha_2^*,\ Y_2 = x_3 + \Omega_*\alpha_3^*$$

enable the system of Eqs.(3.9) to be reduced to the form

$$\begin{gathered} J_1x_1^{\cdot} = (J_2 - J_3)\,(Y_1 - \Omega_*\alpha_2^*)\,(Y_2 - \Omega_*\alpha_3^*) - \\ k\beta_{11}\beta_{12}Y_1 - k\beta_{11}\beta_{13}Y_2 + m_1^* \\ J_2Y_1^{\cdot} = (J_3 - J_1)\,(x_1 + \Omega_*)\,(Y_2 - \Omega_*\alpha_3^*) + \\ k\,(1 - \beta_{12}^2)\, Y_1 - k\beta_{12}\beta_{13}Y_2 + J_2\Omega_*g_2^* + m_2^* \\ J_3Y_2^{\cdot} = (J_1 - J_2)\,(x_1 + \Omega_*)\,(Y_1 - \Omega_*\alpha_2^*) - \\ k\beta_{12}\beta_{13}Y_1 + k\,(1 - \beta_{13}^2)\, Y_2 + J_3\Omega_*g_3^* + m_3^* \end{gathered} \tag{3.11}$$

The required vector function $\mathbf{g}_*$ is determined using a Lyapunov function of the form

$$2V = J_2Y_1^2 + J_3Y_2^2 + p\mathbf{e}_*'\mathbf{e}_* + \gamma^{-1}\boldsymbol{\alpha}_*'\boldsymbol{\alpha}_*,\quad p > 0,\ \gamma > 0 \tag{3.12}$$

The total derivative of V with respect to time, by virtue of Eqs.(3.10) and (3.11), is given by

$$\begin{gathered} V^{\cdot} = k\,(1 - \beta_{12}^2)\, Y_1^2 + k\,(1 - \beta_{13}^2)\, Y_2^2 + [(J_3 - J_2)\,(x_1 + \Omega_*) - \\ 2k\beta_{12}\beta_{13}]\, Y_1Y_2 + J_2\Omega_*Y_1g_2^* + J_3\Omega_*Y_2g_3^* + \\ p\mathbf{e}_*'A_*\mathbf{e}_* + \boldsymbol{\alpha}_*'\,[pB'\,(\partial\mathbf{f}/\partial\xi)'\, B\mathbf{e}_* + \\ \gamma^{-1}\mathbf{g}_* - \boldsymbol{\Psi}\,(Y_1,\ Y_2)] + v\,(Y_1,\ Y_2,\ \boldsymbol{\alpha}_*) \end{gathered} \tag{3.13}$$
$$\boldsymbol{\Psi}\,(Y_1, Y_2) = \{0,\ (J_1 - J_2)\,(x_1 + \Omega_*)\Omega_*Y_2,\ (J_3 - J_1)\,(x_1 + \Omega_*)\,\Omega_*Y_1\}$$

($v\,(Y_1,\ Y_2,\ \boldsymbol{\alpha}_*)$ is a vector function whose expansion in powers of the arguments starts with terms of not less than the third degree).

Specifying the vector function $\mathbf{g}_*$ by the relationship

$$\mathbf{g}_* = -p\gamma B'\left(\frac{\partial\mathbf{f}}{\partial\xi}\right)' B\mathbf{e}_* + \gamma\boldsymbol{\Psi}\,(Y_1, Y_2) \tag{3.14}$$

we replace (3.13) with the expression

$$V^{\cdot} = W\,(Y_1,\ Y_2,\ \mathbf{e}_*) + v\,(Y_1,\ Y_2,\ \boldsymbol{\alpha}_*)$$

where W is a quadratic form in the variables Y_1, Y_2 and $\mathbf{e}_*$.

If we choose the weights in (3.12) so that $W\,(Y_1,\ Y_2,\ \mathbf{e}_*)$ is a negative-define function of its arguments, then by the results of Sect.2 the control

$$M_y = k\,(\omega_y - \eta_2\,(\boldsymbol{\eta}'\boldsymbol{\omega})),\ M_z = k\,(\omega_z - \eta_3\,(\boldsymbol{\eta}'\boldsymbol{\omega})) \tag{3.15}$$

and the identification algorithm (3.14) ensure asymptotic stability of system (3.10), (3.11) with respect to the variables $Y_1, Y_2, \mathbf{e}_*, \boldsymbol{\alpha}_*$. The right-hand side of the first equation in system (3.11) tends to zero as $t \to \infty$ and therefore $x_1 \to c = \text{const}$. The permanent rotation (3.3) of the rigid body is thus Lyapunov stable.

We now have to rewrite the identification algorithm (3.14) in the original xyz basis. Assuming, for simplicity, that $J_2 = J_3 = J_0$ and making the necessary transformations, we rewrite the equation of the identification process in the form

$$\begin{Vmatrix}\dot\eta_2\\ \dot\eta_3\end{Vmatrix} = p\gamma\left(\frac{\partial \mathbf{f}}{\partial \tau}\right)'(\boldsymbol{\omega}-\mathbf{y}) - \gamma(J_0-J_1)\Omega_*\xi'\boldsymbol{\omega}D(\mathbf{X}-\Omega_*\boldsymbol{\alpha}) \tag{3.16}$$

$$D = \begin{Vmatrix}\beta_{21} & 0 & -\beta_{11}\\ -\beta_{21} & \beta_{11} & 0\end{Vmatrix},\quad \eta_1 = (1-\eta_2^2-\eta_3^2)^{1/2}$$

The second term in Eq.(3.16) is approximed by the relation

$$\mathbf{F} = \gamma(J_0-J_1)\Omega_*\eta'\boldsymbol{\omega}\begin{Vmatrix}\eta_3 & 0 & -\eta_1\\ -\eta_2 & \eta_1 & 0\end{Vmatrix}(\boldsymbol{\omega}-\Omega_*\eta) \tag{3.17}$$

The resulting error is expanded in a series in powers of α and $\mathbf{X}$ starting with quadratic terms.

When the deviation of the x axis from x_* is small, the expression for $\mathbf{F}$ is simplified:

$$\mathbf{F} \approx \gamma(J_0-J_1)\Omega_*^2\begin{Vmatrix}-(\omega_z-\Omega_*\eta_3)\\ \omega_y-\Omega_*\eta_2\end{Vmatrix}$$

4. *Example.* For a rigid body with the parameters of the ellipsoid of inertia

$$J_1 = 0.1\times10^n,\quad J_2 = J_3 = 10^n \ (\text{kg}\times\text{m}^2)$$

(n *is an arbitrary integer*) and the unit vector $\xi = \{0.999848, -0.012338, 0.012338\}$ of the principal axis of inertia x_*, Fig.1 shows the variation of the angular velocities ω_x, ω_y, ω_z during the stabilization of its permanent rotation (3.3) ($\Omega = 1\,\text{sec}^{-1}$) by the control algorithm (3.15) with the identification law (3.16)-(3.17).

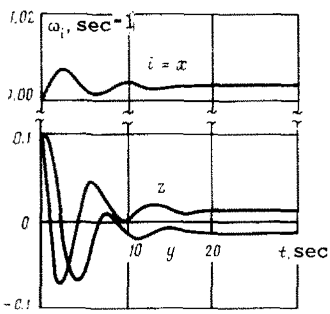

Fig.1

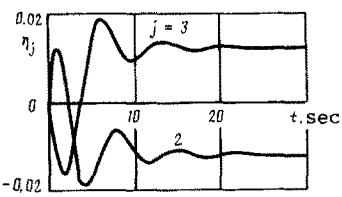

Fig.2

Fig.2 illustrates the dynamics of the process of identifying the vector ξ. Initially, *nothing is known about the vector* $\eta = \{\eta_i\}$ $(i = 1, 2, 3)$ and it is defined in the form $\eta(0) = \{1, 0, 0\}$.

The simulation was carried out for the following parameter values in (3.15)-(3.17): $k = -0.3\times10^n$ Nmsec, $p = 25\times10^n\,\text{kgm}^2$, and $\gamma^{-1} = 5\times10^n\,\text{kgm}^2\,\text{sec}^{-2}$.

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